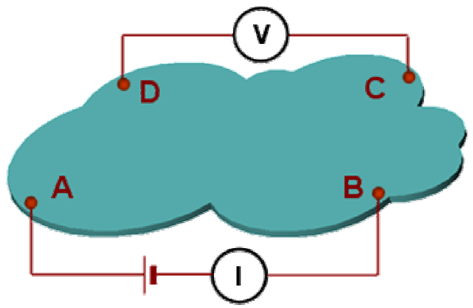
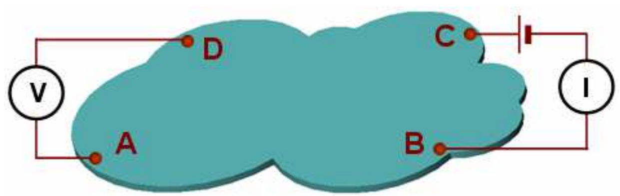
Transport Measuremtents in the van der Pauw Geometry

required: sheet system of (in many cases unknown) thickness d with homogeneous charge carrier density and mobility



measure longitudinal resistance

$$R_{AB,CD} = R_1 = \frac{U_{CD}}{I_{AB}}$$



and measure longitudinal resistance

$$R_{BC,DA} = R_2 = \frac{U_{DA}}{I_{BC}}$$

and get the sheet resistance from the average by

$$R_S = \frac{1}{\sigma_{\square}} = \frac{1}{\sigma\, d} = \frac{\pi}{\ln 2} \cdot \frac{R_1 + R_2}{2} \cdot f\left(\frac{R_1}{R_2}\right)$$

with the weakly warying van der Pauw function f(Q) of the order of 1.

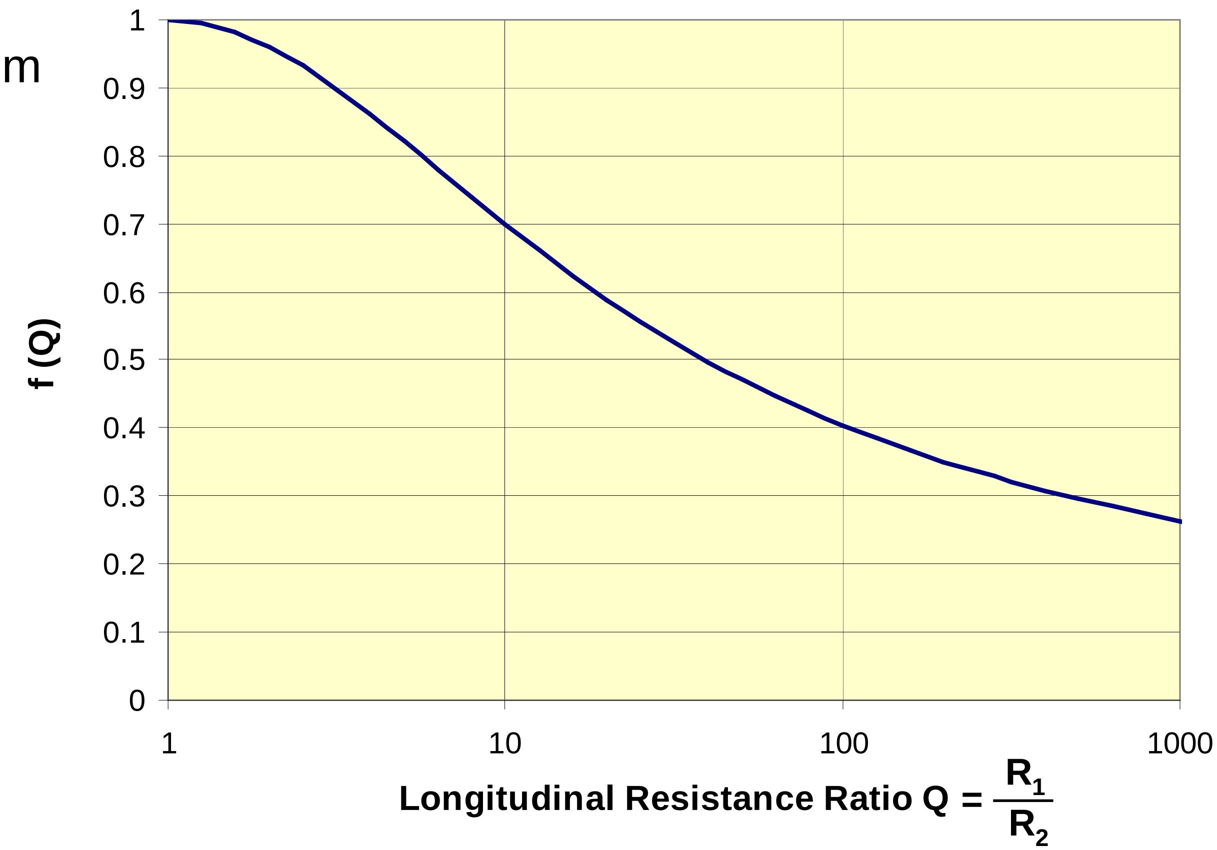
Transport Measuremtents: the van der Pauw Function

Inserting the above Ansatz

$$R_S = \frac{1}{\sigma_\square} = \frac{\pi}{\ln 2} \cdot \frac{R_1 + R_2}{2} \cdot f\left(\tfrac{R_1}{R_2}\right)$$

into van der Pauw's theorem

yields an implicit equation for f(Q) that is easyly solved iteratively and gives



Note:

- Symmetry of R_1 ←→ R_2 guarantees f(1/Q)) f(Q
- f(1)=1
- f drops to 0,7 only even when R_1 = 10 R_2 !

Historic Excursion: Philips Research Labs 1958

A METHOD OF MEASURING THE RESISTIVITY AND HALL COEFFICIENT ON LAMELLAE OF ARBITRARY SHAPE

621.317.331:538.632.083

Resistivity and Hall-coefficient measurements at different temperatures play an important part in research on semiconductors, such as germanium and silicon [1]), for it is from these quantities that the mobility and concentration of the charge carriers are found.

Such measurements are commonly carried out with a test bar as illustrated in *fig. 1*. The resistivity is found directly from the potential difference and the distance between the contacts O and P, the current i and the dimensions of the bar. To determine the Hall coefficient the bar is subjected to a magnetic field B applied at right angles to the direction of the current and to the line connecting the diametrically opposite contacts O and Q. From the potential difference thus produced between these latter contacts the Hall coefficient is derived. (The relation between the Hall coefficient and the change in electric potential distribution due to a magnetic field will be explained presently.)

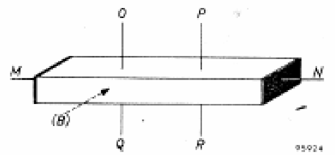


Fig. 1. Classical form of sample used for resistivity and Hall-coefficient measurements. The test bar is provided with current contacts M and N and voltage contacts O, P, Q and R. The fourth voltage contact R serves for check measurements.

In measurements performed at low temperatures — e.g. in liquid nitrogen — point contacts possess resistances of the order of megohms, in consequence of which the voltages cannot be determined with sufficient accuracy. In such cases "bridge-shaped" samples are used as illustrated in *fig. 2*. The voltage and current contacts here have a relatively large surface area, and hence the contact resistances are low.

The methods referred to are based on the fact that the geometry of the sample ensures a simple pattern of virtually parallel current stream-lines. Formulae have been devised to correct for the deviation from parallelism in fig. 2, caused by the finite width of the arms. A drawback of the bridge-shaped sample is that it is rather difficult to make, having to be cut out of the brittle semiconductor material with an ultrasonic tool. There is therefore a considerable risk of breakage, particularly when the arms are made narrow.

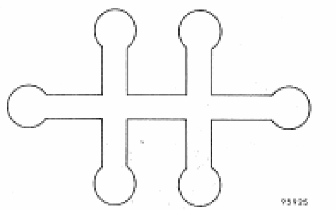


Fig. 2. The bridge-type sample, which is provided with relatively large contact faces to reduce contact resistances. This form is of special importance in measurements at low temperatures.

In the following we shall describe a method of performing resistivity and Hall-coefficient measurements on lamellae of arbitrary shape [2]). The lamella must not, however, contain any (geometrical) holes.

New method of measuring resistivity

We take a flat lamella, completely *free of holes*, and provide it with four *small* contacts, M, N, O and P, at arbitrary places *on the periphery* (*fig. 3*). We apply a current i_{MN} to contact M and take it off at contact N. We measure the potential difference $V_P - V_O$ and define:

$$R_{MN,OP} = \frac{V_P - V_O}{i_{MN}}.$$

Analogously we define:

$$R_{NO,PM} = \frac{V_M - V_P}{i_{NO}}.$$

The new method of measurement is based on the theorem that between $R_{MN,OP}$ and $R_{NO,PM}$ there exists the simple relation:

$$\exp\left(-\frac{\pi d}{\varrho} R_{MN,OP}\right) + \exp\left(-\frac{\pi d}{\varrho} R_{NO,PM}\right) = 1, \quad (1)$$

where d is the thickness of the lamella and ϱ the

[1]) See e.g. C. Kittel, Introduction to solid state physics, 2nd edition, Wiley, New York 1956, Chapter 13, p. 347 et seq.

[2]) L. J. van der Pauw, A method of measuring specific resistivity and Hall effect of discs of arbitrary shape, Philips Res. Repts. **13**, 1-9, 1958 (No. 1).

This expression represents a relation between f and x_1/x_2, and hence also between f and $R_{MN,OP}/R_{NO,PM}$ (see 5). The relation is shown graphically in fig. 5. By re-writing (8) to give ϱ and substituting for x_1 and x_2 from (5), we find formula (4).

Method of measuring the Hall coefficient

The Hall coefficient, too, can be measured on an arbitrary lamella as in fig. 3. We then apply the current to one of the contacts, say M, and take it off at the contact following the succeeding one, i.e. in our case at O. We measure $R_{MO,NP}$, after which we set up an uniform magnetic induction B at right angles to the surface of the lamella. This changes $R_{MO,NP}$ by an amount $\Delta R_{MO,NP}$. We shall now denote the Hall coefficient R_H and show that it is given by:

$$R_H = \frac{d}{B}\,\Delta R_{MO,NP}, \quad \ldots \quad (9)$$

provided that:

a) the contacts are sufficiently small,
b) the contacts are on the periphery,
c) the lamella is of uniform thickness and free of holes.

The validity of formula (9) depends on the distribution of current stream-lines not changing when the magnetic field is applied. With samples of the classical shape of figs. 1 and 2, where the current stream-lines are always parallel to the edges of the sample, there is evidently no change. From the properties of the vector field representing the current density it follows that the same also applies to lamellae of arbitrary shape, provided the above conditions are satisfied [3]).

Under the magnetic induction B, the charge carriers, with charge q, are subjected to a force perpendicular to the stream-lines and perpendicular to the lines of magnetic induction. The magnitude of this force is $F = qvB$, where v is the velocity of the charge carriers. Between v, the concentration n of the charge carriers and the current density J there exists the relation $v = J/nq$. Dividing the force exerted on the charge carriers by their charge q, we see that the effect of the magnetic field is equivalent to an apparent electric field E_H, the Hall electric field, for which we can write [4]):

$$E_H = \frac{1}{nq} J B.$$

[3]) The proof of this statement is also indicated in the paper quoted under [2]).

[4]) This formula is not entirely exact because, apart from their ordered motion with velocity v, the electrons also move randomly owing to thermal agitation. More precise calculation shows, however, that the formula given here is a good approximation.

E_H is proportional to J and to B; the proportionality constant ($= 1/nq$) is called the Hall coefficient R_H.

Since q is known, one can calculate from R_H the concentration n of the charge carriers.

The fact that the current stream-lines are not affected by the magnetic field implies that after application of the magnetic field the electric field is no longer in the same direction as the current stream-lines, but has acquired a transverse component E_t which exactly compensates the

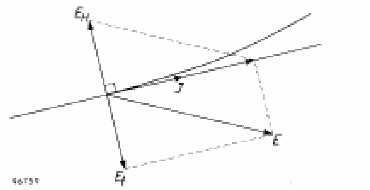


Fig. 8. The resultant of the electrical field-strength E and the Hall field-strength E_H lies in the direction of the current density J. Resolving E in directions perpendicular and parallel to J therefore yields a perpendicular component E_t which in magnitude is equal to E_H.

apparent Hall electric field E_H (*fig. 8*). The change $\Delta(V_P - V_N)$ in the potential difference between P and N is therefore given by integrating E_t from P over a path orthogonal to the current stream-lines to N' across the lamella (*fig. 9*), and thence along the periphery — i.e. along a stream-line — from N' to N. The last portion of the path makes no contribution to the integral; hence

$$\Delta(V_P - V_N) = \int_P^{N'} E_H\,\mathrm{d}s = R_H B \int_P^{N'} J\,\mathrm{d}s = R_H B \frac{i_{MO}}{d},$$

where d is again the thickness of the lamella. This expression leads directly to (9).

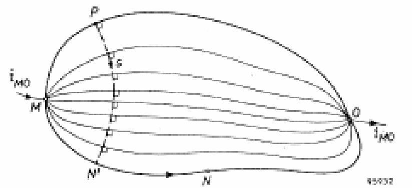
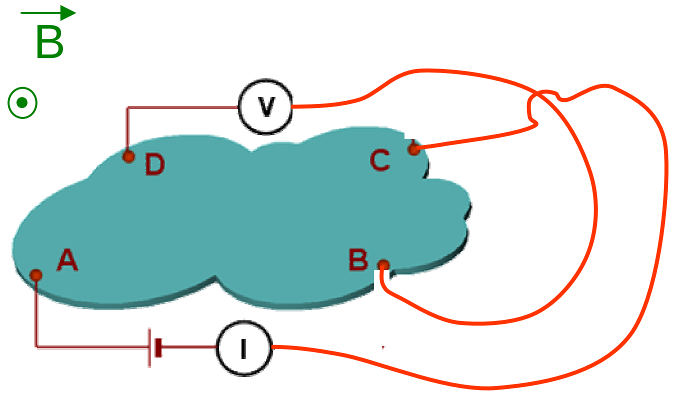


Fig. 9. To calculate by how much the potential difference between points P and N changes when a magnetic field is applied at right-angles to the plane of the sample, the transverse electric field E_t produced by the magnetic field is integrated along the path s which runs from P, orthogonal to the current stream-lines, to N' and thence along the periphery from N' to N.

Measuring the Hall Effect in the van der Pauw Geometry



measure the transverse resistance

$$R_{AC,BD} = R_1^T = \frac{U_{BD}}{I_{AC}}$$

or measure transverse resistance

$$R_{BD,CA} = R_2^T = \frac{U_{CA}}{I_{BD}}$$

with and without magnetic field B $\perp$ sheet.

→ The experimental Hall constant $\tilde{R}_H$ is then

$$\tilde{R}_H = \frac{\Delta U_{BD}}{B \cdot I_{AC}} = \frac{R_1^T(B) - R_1^T(B=0)}{B} = \frac{\Delta U_{CA}}{B \cdot I_{BD}} = \frac{R_2^T(B) - R_2^T(B=0)}{B}$$

By convention, the Hall constant R_H is defined as $R_H = \tilde{R}_H \cdot d$

For **unipolar** transport, i.e.only electrons or only holes with density c:

$$\tilde{R}_H = \frac{r}{q\,(c \cdot d)} = \frac{r}{q\,c_\square}$$ ← areal, i.e. depth-integrated charge carrier density, [...] = cm $^{-2}$!

$$R_H = \frac{r}{q \cdot c}$$

with the charge q= +/-e of the charge carriers and the Hall scattering factor r of the order of 1

The Hall Scattering Factor

The inelasting scattering time of electrons and holes always follows a Poisson statistics with distribution function $f_\tau(t) = \frac{1}{\tau}\exp\left(-\frac{t}{\tau}\right)$.

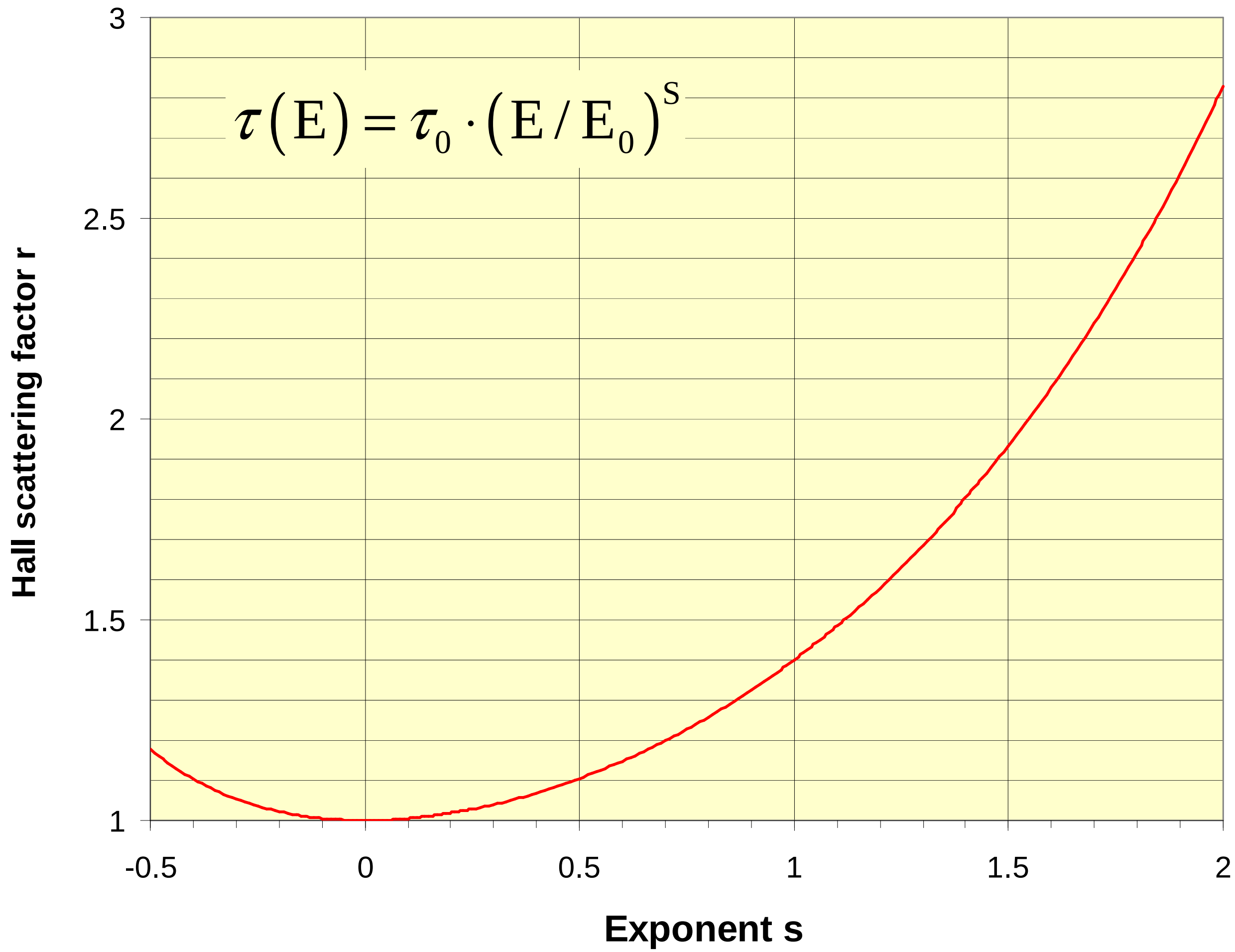
The parameter $\tau(v^2)=\tau(E)$ in general depends on the kinetic energy of the charge carriers.

The Hall scattering factor r is

$$r = \frac{\left\langle \tau^2 \right\rangle_E}{\left\langle \tau \right\rangle_E^2}$$

where < > stands for averaging over the energy distribution of the charge carriers.

In most cases $\tau(E)$ follows a power law function where the exponent is determinded by the dominating scattering mechanism.



For most cases -0,5 < s < 1,5 and thus 1< r < 2, i.e. the scattering mechanism has only a **weak influence** on the determination of the Hall mobilities!

Combining Hall Effect and Conductivity Measurement

For **unipolar** transport, i.e.only electrons or only holes with density c:

$$\sigma_{\square} = \frac{1}{R_S} = e\,(c \cdot d)\,\mu = e\,c_{\square}\,\mu \qquad \text{and} \qquad \tilde{R}_H = \frac{r}{q \cdot c_{\square}}$$

sheet charge carrier density [cm^{-2}]

→ $$c_{\square} = \frac{r}{q \cdot |\tilde{R}_H|}$$ yields areal c.c. density (up to r~1)

→ $$\tilde{R}_H \cdot \sigma_{\square} = \frac{\tilde{R}_H}{R_S} = \text{sign}(q) \cdot \mu \cdot r$$ yields sign and mobility of the c.c. (up to r~1)

→ For unipolar transport, the **combination** of conductivity measurement and Hall effect gives sign, areal density and mobility of the charge carriers (up to r~1) !

Note:

- Conductivity and Hall effect are insensitive w.r.t. the sample thickness d
- Only when d is known independently, c and σ can be evaluated from $c_{\square}$ and $\sigma_{\square}$.

Combining Hall Effect and Conductivity Measurement

For **bipolar** transport, i.e.only electrons and holes with density:

$$\sigma_{\square} = \frac{1}{R_S} = e\left(\mu_e n + \mu_h p\right)$$

and

$$\tilde{R}_H = \frac{\mu_h{}^2 r_h p_{\square} - \mu_e{}^2 r_e n_{\square}}{e\left(\mu_h p_{\square} - \mu_e n_{\square}\right)^2}$$

→ In general, two data for 4 unknown (+ r) → not conclusive!

For the special case $r_e = r_h$ and $n_{\square} = p_{\square} = c_{\square}$:

$$\sigma_{\square} = \frac{1}{R_S} = e\, c_{\square}\left(\mu_e + \mu_h\right)$$

and

$$\tilde{R}_H = \frac{r}{e c_{\square}} \frac{\mu_h - \mu_e}{\mu_h + \mu_e}$$

→ $$c_{\square}\left(\mu_e + \mu_h\right) = \frac{1}{e\, R_S}$$ yields areal density times sum of the mobililties

→ $$\tilde{R}_H \cdot \sigma_{\square} = \frac{\tilde{R}_H}{R_S} = \left(\mu_h - \mu_e\right) \cdot r$$ yields the difference of the mobililties

(up to r~1)

→ Still not conclusive without further input, except for $\mu_h \ll \mu_e$ or $\mu_e \ll \mu_h$.